Generating mutual coherence from incoherence with the help of a phase-conjugate mirror

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Although the signal and idler light beams produced in the process of spontaneous parametric down-conversion are mutually incoherent and exhibit no one-photon interference, interference effects should be observable after either the signal or the idler is reflected by a phase-conjugate mirror. The degree of coherence can, in principle, be as large as unity. We refer to the relationship between the signal and the idler produced by down-conversion as anticoherent and introduce a natural measure for the degree of anticoherence.

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I. INTRODUCTION

The process of spontaneous, parametric down-conversion has served as the source of photons for numerous experiments in recent years. In this process a pump photon, incident on a nonlinear crystal, in effect splits into two photons, usually known as the signal and the idler photon, which are highly correlated in time [1–3] and emerge in an entangled quantum state. The high rate at which photon pairs can be detected in coincidence leads to a large violation of classical probability [4,5]. The down-conversion process has also been used to demonstrate certain quantum interference effects, both with one and with two down-converters [6–10].

On the other hand, direct one-photon (or second-order) interference between the signal and the idler has never been observed in spontaneous down-conversion; the signal and idler beams always behave as mutually incoherent. We wish to draw attention to the fact that the emerging two light beams can, however, be made mutually coherent and to exhibit interference after one or the other is reflected from a phase-conjugate mirror. Hence mutual coherence can be established between two light beams that start off as mutually incoherent.

II. THEORY

We take the energy for the parametric interaction at the down-converter between the signal, the idler, and the pump, each of which is treated as monochromatic for simplicity, to be given by

$$\hat{H}_I=\hbar g[v_0 e^{-2i\omega t}\hat{a}_s^\dagger(t)a_i^\dagger(t)+\text{H.c.}], \tag{1}$$

where ω is the frequency of both the signal and the idler. For simplicity, we treat the strong coherent pump field as classical and of complex amplitude $v_0 e^{-2i\omega t}$ and only the down-converted fields $\hat{a}_s,\hat{a}_i$ are quantized. The coupling constant g, which depends on the nonlinear susceptibility of the down-converter, is assumed to be real. If the initial quantum state of the down-converted field is the vacuum, then the state $|\psi(t)\rangle$ in the interaction picture after a time t that is short compared with the mean time interval between down-conversions, but long compared with the coherence time of the down-converted light, may be written [7]

$$|\psi(t)\rangle=m|\text{vac}\rangle_{s,i}-igtv_0|1\rangle_s|1\rangle_i. \tag{2}$$

We have made the assumption that $|gtv_0|\ll 1$, $|m|\approx 1$, so that terms with more than two photons make a very small contribution and may be neglected.

If the signal and idler waves of complex amplitudes $\hat{a}_s$ and $\hat{a}_i$ come together and mix, then the expectation of the intensity of the resultant field is given by

$$\langle\hat{I}\rangle=\langle\psi(t)|(\hat{a}_s^\dagger e^{i\theta_s}+\hat{a}_i^\dagger e^{i\theta_i})(\hat{a}_s e^{-i\theta_s}+\hat{a}_i e^{-i\theta_i})|\psi(t)\rangle, \tag{3}$$

in which θ_s and θ_i are phase shifts that characterize the propagation from source to detector. From Eqs. (2) and (3) we readily find for the resultant light intensity

$$\langle\hat{I}\rangle=2|gtv_0|^2. \tag{4}$$

This is just the sum of signal and idler intensities and evidently exhibits no interference. The absence of mutual coherence between the signal and the idler is expressed more explicitly by the equation

$$\langle\psi(t)|\hat{a}_s^\dagger\hat{a}_i|\psi(t)\rangle=0=\langle\psi(t)|\hat{a}_i^\dagger\hat{a}_s|\psi(t)\rangle \tag{5}$$

and it can be regarded as a reflection of the fact that the field in a two-photon state has a completely indefinite phase.

Now consider the experimental situation illustrated in Fig. 1. Both the signal s_1 and the idler i_1 of frequency ω produced by down-conversion in the nonlinear crystal are reflected back on themselves and are then labeled s_2 and i_2. However, whereas the signal is reflected by an ordinary mirror M of reflectivity $\mathscr{R}$, the idler is reflected by the phase-conjugate mirror (PCM). v_s and v_i label the vacuum modes entering from the outside at the mirror M and at the PCM, respectively. The PCM is optically pumped by the two strong, coherent pump fields of complex amplitudes v_1,v_2 of fre-

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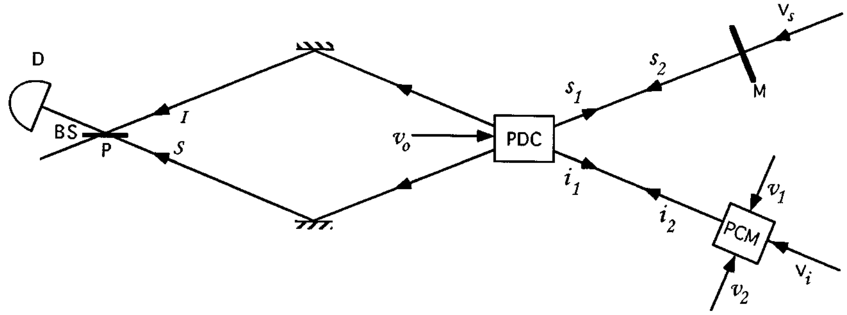

FIG. 1. Outline of the experiment for generating mutual coherence after reflection of one beam by a phase conjugate mirror. PDC, parametric down-converter crystal; PCM, phase-conjugate mirror; BS, beam splitter; M, mirror; D, photodetector; s_1 and i_1, the outgoing signal and idler modes produced by the PDC; s_2 and i_2, the corresponding incoming field modes after reflection.

quency ω, which, like v_0, are treated classically. After traversing the crystal PDC in the reverse direction, the two reflected waves of amplitudes denoted by $\hat{a}_{s_2}$ and $\hat{a}_{i_2}$ are allowed to come together at P, perhaps with the help of a 50%:50% beam splitter located at P, and interfere.

We begin by relating the wave amplitudes at P to those of the light emitted by the parametric down-converter (PDC). If the fields are all monochromatic and of frequency ω, and if the amplitudes $\hat{a}_{s_1},\hat{a}_{s_2}$ refer to outgoing and incoming fields at the position of the down-converter, respectively, then [11]

$$\hat{a}_{s_2}=(\mathscr{R}\hat{a}_{s_1}e^{-i\omega\tau_s}+\mathscr{T}'\hat{a}_{v_s})e^{-i\omega\tau_s}, \tag{6}$$

where τ_s is the propagation time between the PDC and the mirror M. $\mathscr{R}$ and $\mathscr{T}$ are the reflectivity and transmissivity of the mirror M from one side and $\mathscr{R}'$ and $\mathscr{T}'$ from the other side. $\hat{a}_{v_s}$ is a vacuum mode amplitude representing the field entering from the outside at M. If τ_s' is the propagation time of the reflected signal wave from the PDC to P, then the reflected signal at P is given by

$$\hat{a}_S=\hat{a}_{s_2}e^{-i\omega\tau_s'}=(\mathscr{R}\hat{a}_{s_1}e^{-i\omega\tau_s}+\mathscr{T}'\hat{a}_{v_s})e^{-i\omega(\tau_s+\tau_s')}. \tag{7}$$

We now relate the idler and the reflected idler waves in a similar manner. From the known input-output relations of the PCM we have [11–13]

$$\hat{a}_{i_2}=\hat{a}_{v_i}e^{-i\omega\tau_i}\sec(KL)-ie^{i(\theta_1+\theta_2)}\tan(KL)\hat{a}^\dagger_{i_1}, \tag{8}$$

where $\hat{a}_{v_i}$ represents the vacuum field entering the PCM and $\theta_1=\arg v_1$ and $\theta_2=\arg v_2$. The τ_i,τ_i' are idler propagation times from the PDC to the PCM and from the PDC to P. L is the length of the PCM medium and K is given by

$$K=G|v_1v_2|V. \tag{9}$$

Here G is the mode coupling constant of the PCM and V is the wave velocity in the medium of the PCM. From Eq. (8) it then follows that the reflected idler wave at P is given by

$$\hat{a}_I=\hat{a}_{i_2}e^{-i\omega\tau_i'}=[\hat{a}_{v_i}\sec(KL) -ie^{i(\theta_1+\theta_2)}\tan(KL)\hat{a}^\dagger_{i_1}e^{i\omega\tau_i}]e^{-i\omega(\tau_i+\tau_i')}. \tag{10}$$

The equation has been written so that $\hat{a}_{i_1}$ refers to the idler field at the PDC.

As always, the mutual coherence between the outgoing fields $\hat{a}_S$ and $\hat{a}_I$ is measured by the cross correlation $\langle\psi(t)|\hat{a}^\dagger_S\hat{a}_I|\psi(t)\rangle$ and with the help of Eqs. (2) and (6)–(10) we obtain

$$\begin{aligned}\langle\psi(t)|\hat{a}^\dagger_S\hat{a}_I|\psi(t)\rangle&=[m^*\langle\text{vac}|+igtv_0^*\,{}_{s_1}\langle1|_{i_1}\langle1|_{v_i}\langle0|_{v_s}\langle0|][\mathscr{R}^*\hat{a}^\dagger_{s_1}e^{i\omega\tau_s}+\mathscr{T}'^*\hat{a}^\dagger_{v_s}]e^{+i\omega(\tau_s+\tau_s')}\\&\quad\times[\hat{a}_{v_i}\sec(KL)-ie^{i(\theta_1+\theta_2)}\tan(KL)e^{i\omega\tau_i}\hat{a}^\dagger_{i_1}]e^{-i\omega(\tau_i+\tau_i')}[m|\text{vac}\rangle-igtv_0|1\rangle_{s_1}|1\rangle_{i_1}|0\rangle_{v_s}|0\rangle_{v_i}]\\&=mgtv_0^*e^{i(\theta_1+\theta_2)}\mathscr{R}^*e^{i\omega(2\tau_s+\tau_s'-\tau_i')}\tan(KL).\end{aligned} \tag{11}$$

This is nonzero in general and depends on the optical path length. It also depends on the existence of a vacuum contribution to the down-converted state, as can be seen by the appearance of the coefficient m which is close to unity, but not on the vacuum fields entering at M and at the PCM. $|\text{vac}\rangle$ is shorthand for $|0\rangle_{s_1}|0\rangle_{i_1}|0\rangle_{v_s}|0\rangle_{v_i}$. It follows that, after the idler is reflected by the PCM, there is generally mutual coherence between the signal and the reflected idler.

In order to calculate the degree of coherence, or the visibility, of the interference pattern, we require the average photon numbers in modes S and I. From Eqs. (2) and (7) we have

$$\langle\hat{n}_S\rangle=\|\hat{a}_S|\psi(t)\rangle\|^2 \tag{12}$$

$$=|gtv_0|^2|\mathscr{R}|^2 \tag{13}$$

and from Eqs. (2) and (10) we obtain

$$\begin{aligned}\langle \hat{n}_I\rangle &= \left|\hat{a}_I|\psi(t)\rangle\right|^2\\ &= \left|\tan(KL)[-im|1\rangle_{i_1} - \sqrt{2}gtv_0|1\rangle_{s_1}|2\rangle_{i_1}]\right|^2\\ &= \tan^2(KL)[|m|^2 + 2|gtv_0|^2].\end{aligned}$$

As $|m|$ we assumed to be very nearly unity, and $|gtv_0| \ll 1$, this result can be simplified to

$$\langle \hat{n}_I\rangle \approx \tan^2(KL). \tag{14}$$

From Eqs. (12)–(14) the degree of coherence $|\gamma|$ between the signal and the reflected idler is given by

$$|\gamma| = \frac{|\langle \hat{a}_S^\dagger \hat{a}_I\rangle|}{\langle \hat{n}_S\rangle^{1/2}\langle \hat{n}_I\rangle^{1/2}} = 1. \tag{15}$$

The visibility $\mathscr{V}$ of the interference pattern, on the other hand, is given by

$$\mathscr{V} = \frac{2|\langle \hat{a}_S^\dagger \hat{a}_I\rangle|}{\langle \hat{n}_S\rangle + \langle \hat{n}_I\rangle} = \frac{2|gtv_0||\mathscr{R}||\tan(KL)|}{|gtv_0|^2|\mathscr{R}|^2 + \tan^2(KL)}. \tag{16}$$

This is less than 1 in general, but it can become unity if $|gtv_0||\mathscr{R}| = |\tan(KL)|$.

III. DISCUSSION

The reason for the appearance of mutual coherence after reflection of the idler by a PCM is most readily understood in terms of the phase relationship between the signal and the idler in the process of parametric down-conversion. Whereas the phases of mutually coherent light beams are usually equal or nearly equal, so that the phase difference is constant, it is the sum of the phases of the signal and idler beams produced by down-conversion that is determined. It follows that phase reversal of either the signal or idler creates the conditions for mutual coherence.

This suggests that it may be useful to introduce yet another measure of phase correlation between two fields, besides mutual coherence. In order to illustrate the idea we start with an example from the classical domain. Let $V_1(t)$ and $V_2(t)$ represent two independent complex random processes, such as two optical fields. Let

$$\begin{aligned}V_1(t) &= Ae^{-i[\omega t + \phi_1(t)]},\\ V_2(t) &= Be^{-i[\omega t + \phi_2(t)]},\end{aligned} \tag{17}$$

in which A, B, ω are constants, while each of the phases $\phi_1(t), \phi_2(t)$ independently performs a one-dimensional random walk (Wiener process) in time. From the well-known properties of the Wiener process we have

$$\langle e^{i\phi_j(t)}\rangle = 0 \quad (j=1,2), \tag{18}$$

$$\langle e^{i\phi_j(t+\tau) - i\phi_j(t)}\rangle = e^{-D^2\tau^2} \quad (j=1,2), \tag{19}$$

$$\langle e^{i\phi_j(t+\tau) + i\phi_j(t)}\rangle = 0 \quad (j=1,2), \tag{20}$$

where D is the diffusion constant. Now if $\phi_1(t)$ and $\phi_2(t)$ are independent Wiener processes, then from Eq. (18) we obtain for the cross correlation

$$\Gamma_{12}^{(1,1)}(t,t+\tau) = \langle V_1^*(t)V_2(t+\tau)\rangle = \langle V_1^*(t)\rangle\langle V_2(t+\tau)\rangle = 0, \tag{21}$$

so that the fields $V_1(t)$ and $V_2(t)$ defined by Eqs. (17)–(20) are mutually incoherent.

By contrast, let us consider the two fields

$$\begin{aligned}V_3(t) &= Ae^{-i[\omega t + \phi_1(t)]},\\ V_4(t) &= Be^{-i[\omega t - \phi_1(t)]},\end{aligned} \tag{22}$$

which are strongly correlated. However,

$$\begin{aligned}\Gamma_{34}^{(1,1)}(t,t+\tau) &= \langle V_3^*(t)V_4(t+\tau)\rangle\\ &= ABe^{-i\omega\tau}\langle e^{i[\phi_1(t+\tau)+\phi_1(t)]}\rangle = 0\end{aligned} \tag{23}$$

by reason of Eq. (20), so that $V_3(t)$ and $V_4(t)$ are also mutually incoherent. Yet obviously these two cases are significantly different.

We shall adopt the terminology that $V_3(t)$ and $V_4(t)$ given by Eqs. (22) are mutually *anticoherent*. In order to include both classical and quantum fields in the definition of anticoherence, we introduce the normally ordered correlation function of order N,M:

$$\begin{aligned}\Gamma^{(N,M)}(x_1,\ldots,x_N;y_M,\ldots,y_1) \equiv \langle \hat{E}^{(-)}(x_1)\cdots\hat{E}^{(-)}(x_N)\\ \times \hat{E}^{(+)}(y_M)\cdots\hat{E}^{(+)}(y_1)\rangle,\end{aligned} \tag{24}$$

where $\hat{E}^{(+)}(x)$ and $\hat{E}^{(-)}(x)$ are the positive frequency part and the negative frequency part of the Hermitian field $\hat{E}(x)$ and x,y are any space-time points. Setting $\hat{E}^{(+)}(x_r) \equiv \hat{E}_r^{(+)}$, we describe two fields $\hat{E}_1, \hat{E}_2$ as being mutually anticoherent to some extent if

$$\langle \hat{E}_1^{(-)}\hat{E}_2^{(-)}\rangle = \Gamma^{(2,0)}(x_1,x_2) \neq 0. \tag{25}$$

If two fields are partly anticoherent and if one of them is reflected by a phase-conjugate mirror, the resulting two fields are partly coherent.

Sometimes it is convenient also to associate a degree of anticoherence with the field. From the Schwarz inequality we have

$$|\Gamma^{(2,0)}(x_1,x_2)|^2 \leq \Gamma^{(2,2)}(x_1,x_2;x_2,x_1), \tag{26}$$

and this suggests that we define the *degree of anticoherence* $|\gamma^{(2,0)}|$ via the normalized correlation

$$|\gamma^{(2,0)}| \equiv \frac{|\Gamma^{(2,0)}(x_1,x_2)|}{[\Gamma^{(2,2)}(x_1,x_2;x_2,x_1)]^{1/2}}. \tag{27}$$

Then $0 \leq |\gamma^{(2,0)}| \leq 1$ by definition from the Schwarz inequality.

Let us now apply these definitions to the down-converted signal and idler fields $\hat{a}_s, \hat{a}_i$. With the help of the quantum state $|\psi(t)\rangle$ given by Eq. (2) we readily obtain

$$|\gamma^{(2,0)}| = \frac{|\langle \hat{a}_s^\dagger \hat{a}_i^\dagger \rangle|}{\langle \hat{a}_s^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_s \rangle^{1/2}} = \frac{|igtv_0^* m|}{|gtv_0|} = 1, \qquad (28)$$

since $|m| \approx 1$. It follows that the signal and idler fields produced by down-conversion are completely anticoherent. However, after the idler is reflected by a phase-conjugate mirror, the signal and reflected idler fields are mutually coherent.

It should be possible to demonstrate this phenomenon experimentally. The principal experimental problem is probably connected with the need for the PCM pump beam frequency to be exactly one-half the frequency of the pump beam for the down-converter. Alternatively, one can argue that the effect has already been observed in interference experiments with two down-converters [9], because of the similarity between down-conversion and phase conjugation.

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