Violations of locality in polarization-correlation measurements with phase shifters

J. R. Torgerson, D. Branning, C. H. Monken, and L. Mandel
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 27 December 1994)

Two-photon polarization-correlation measurements have been performed in order to test local realism under conditions in which the two photons pass through quarter-wave plates before reaching the polarizers. The motivation for this experiment is the fact that two previous experiments with quarter-wave plates preceding the polarizers failed to observe statistically significant violations of locality [J. F. Clauser, Nuovo Cimento **33B**, 740 (1976); A. J. Duncan, H. Kleinpoppen, and Z. A. Sheikh, in *Bell's Theorem and the Foundations of Modern Physics*, edited by A. van der Merwe, F. Selleri, and G. Tarozzi (World Scientific, Singapore, 1992), p. 161]. We find that the Clauser-Horne type of Bell inequality is violated by about 40 standard deviations. Possible reasons for the failure of earlier experiments are discussed.

PACS number(s): 03.65.Bz, 12.20.Fv

I. INTRODUCTION

Violations of locality, or local realism, in two-photon polarization-correlation experiments have now been demonstrated many times with different sources, usually by showing that a Bell inequality is violated, and the existence of the phenomenon is hardly in doubt [1–6]. There are, however, at least two published examples of experiments whose results are somewhat strange and appear to contradict the foregoing [7,8]. One is an experiment by Clauser in which a mercury atomic cascade served as a two-photon source [7], and the other is an experiment by Duncan, Kleinpoppen, and Sheikh based on the two-photon decay of metastable hydrogen [8]. Both experiments are distinguished by the fact that each photon passes through a quarter-wave plate before encountering a linear polarizer and a photodetector, and neither experiment results in a clear violation of a Bell inequality. Below, we present an analysis of an experiment of this type, which suggests that the anomalous results may be attributable to imperfections in the quarter-wave plates that were used to produce circular polarization, as was also suggested by Clauser.

We have recently reported on a polarization-correlation experiment with a combination of down converter and beam splitter serving as a two-photon source [9,10], in which local realism was found to be violated. By inserting a circular polarizer in each photon channel before the linear polarizer and photodetector, we are in a position to more or less repeat the interference experiments of Clauser and of Duncan, Kleinpoppen, and Sheikh. We present the results of such an experiment. Unlike previous workers, we find an unambiguous violation of locality by about 40 standard deviations in the experiment.

II. THEORY OF THE EXPERIMENT

We consider the idealized experimental situation illustrated in Fig. 1. Two signal and idler photons emitted simultaneously in the process of parametric down conversion from a nonlinear crystal of $LiIO_3$, which is optically pumped in the uv, emerge with similar polarizations from the crystal with type *I* phase matching. An optical rotator R inserted in the idler beam makes the two polarizations orthogonal. The two photons impinge from opposite sides on the beam splitter BS at near normal incidence and mixed beams leave the BS almost in opposite directions in arms 1 and 2. Each of the two beams passes first through a phase plate Q that can be used to produce circular polarizations, then through a linear polarizer P set to some adjustable polarization angle θ, and then to a photodetector D.

It has been shown [11] that the state of the photon pair just past the BS ideally can be expressed in the form

$$|\Psi\rangle = \mathcal{T}_x\mathcal{T}'_y|1\rangle_{1x}|1\rangle_{2y} + \mathcal{R}_x\mathcal{R}'_y|1\rangle_{1y}|1\rangle_{2x} + \mathcal{T}_x\mathcal{R}'_y|1\rangle_{1x}|1\rangle_{1y} + \mathcal{R}_x\mathcal{T}'_y|1\rangle_{2x}|1\rangle_{2y} \; . \quad (1)$$

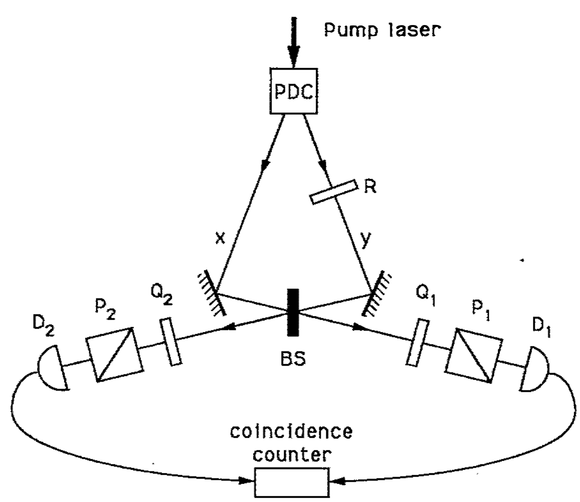

FIG. 1. Outline of the experiment for testing local realism by polarization-correlation measurements.

Here $|1\rangle_{j\lambda}$ is a one-photon state of polarization λ in arm j ($j=1,2$ and $\lambda=x,y$), $\mathcal{R}_\lambda \mathcal{T}_\lambda$ are the complex reflectivity and transmissivity of the beam splitter for λ-polarized light incident from one direction and $\mathcal{R}'_\lambda \mathcal{T}'_\lambda$ from the other. If $\hat{a}_{1x},\hat{a}_{1y}$ are photon annihilation operators for the x-polarization component and the y-polarization component of the field in arm 1, and $\hat{a}_{2x},\hat{a}_{2y}$ are the corresponding operators in arm 2, and if the phase plates are rotated through angles χ_1,χ_2 relative to the (x,y) coordinate systems for the two arms, then the fields emerging from the two phase plates can be represented by

$$\hat{\mathcal{A}}_{1X}=(\hat{a}_{1x}\cos\chi_1+\hat{a}_{1y}\sin\chi_1)e^{i\phi_{1X}} , \tag{2}$$

$$\hat{\mathcal{A}}_{1Y}=(-\hat{a}_{1x}\sin\chi_1+\hat{a}_{1y}\cos\chi_1)e^{i\phi_{1Y}} , \tag{3}$$

$$\hat{\mathcal{A}}_{2X}=(\hat{a}_{2x}\cos\chi_2+\hat{a}_{2y}\sin\chi_2)e^{i\phi_{2X}} , \tag{4}$$

$$\hat{\mathcal{A}}_{2Y}=(-\hat{a}_{2x}\sin\chi_2+\hat{a}_{2y}\cos\chi_2)e^{i\phi_{2Y}} , \tag{5}$$

where the Φ's are any phase shifts characterizing the two phase plates.

Finally, if θ_1,θ_2 are the polarization angles corresponding to the settings of the two (perfect) linear polarizers, then the fields falling on the two photodetectors can be represented by

$$\hat{A}_1=\hat{\mathcal{A}}_{1X}\cos(\theta_1-\chi_1)+\hat{\mathcal{A}}_{1Y}\sin(\theta_1-\chi_2) , \tag{6}$$

$$\hat{A}_2=\hat{\mathcal{A}}_{2X}\cos(\theta_2-\chi_2)+\hat{\mathcal{A}}_{2Y}\sin(\theta_2-\chi_2) . \tag{7}$$

We may now calculate the joint probability $\mathcal{P}_{12}$ of two photon detections by the two photodetectors, given the presence of one photon in each arm. If transmission or reflection at the beamsplitter is a random process, then

$$\mathcal{P}_{12}(\theta_1,\theta_2)=\alpha_1\alpha_2\langle\psi|\hat{A}_1^\dagger\hat{A}_2^\dagger\hat{A}_2\hat{A}_1|\psi\rangle/(|\mathcal{T}_x|^2|\mathcal{T}_y|^2+|\mathcal{R}_x|^2|\mathcal{R}_y|^2) , \tag{8}$$

where the denominator is the probability of one photon in each arm according to Eq. (1), and α_1,α_2 are the two detector quantum efficiencies. From Eqs. (2) to (8) we readily obtain

$$\begin{aligned}\mathcal{P}_{12}(\theta_1,\theta_2)&=\alpha_1\alpha_2|\hat{A}_2\hat{A}_1|\psi\rangle|^2/(|\mathcal{T}_x|^2|\mathcal{T}_y|^2+|\mathcal{R}_x|^2|\mathcal{R}_y|^2)\\
&=\alpha_1\alpha_2\Big|\mathcal{T}_x\mathcal{T}'_y[\cos(\theta_1-\chi_1)\cos(\theta_2-\chi_2)\cos\chi_1\sin\chi_2e^{i(\phi_{1X}+\phi_{2X})}-\sin(\theta_1-\chi_1)\sin(\theta_2-\chi_2)\sin\chi_1\cos\chi_2e^{i(\phi_{1Y}+\phi_{2Y})}\\
&\qquad+\cos(\theta_1-\chi_1)\sin(\theta_2-\chi_2)\cos\chi_1\cos\chi_2e^{i(\theta_{1X}+\phi_{2Y})}\\
&\qquad-\sin(\theta_1-\chi_1)\cos(\theta_2-\chi_2)\sin\chi_1\sin\chi_2e^{i(\phi_{1Y}+\phi_{2X})}]\\
&\quad+\mathcal{R}_x\mathcal{R}'_y[\cos(\theta_1-\chi_1)\cos(\theta_2-\chi_2)\sin\chi_1\cos\chi_2e^{i(\phi_{1X}+\phi_{2X})}-\sin(\theta_1-\chi_1)\sin(\theta_2-\chi_2)\cos\chi_1\sin\chi_2e^{i(\phi_{1Y}+\phi_{2Y})}\\
&\qquad-\cos(\theta_1-\chi_1)\sin(\theta_2-\chi_2)\sin\chi_1\sin\chi_2e^{i(\phi_{1X}+\phi_{2Y})}\\
&\qquad+\sin(\theta_1-\chi_1)\cos(\theta_2-\chi_2)\cos\chi_1\cos\chi_2e^{i(\phi_{1Y}-\phi_{2X})}]\Big|^2/(|\mathcal{T}_x|^2|\mathcal{T}_y|^2+|\mathcal{R}_x|^2|\mathcal{R}_y|^2) .\end{aligned} \tag{9}$$

With the help of the reciprocity relations for a beam splitter [12],

$$\begin{aligned}&\mathcal{R}'_y\mathcal{T}_y^{\prime *}+\mathcal{R}_y^*\mathcal{T}_y=0\\ &|\mathcal{T}'_y|=|\mathcal{T}_y|,|\mathcal{R}'_y|=|\mathcal{R}_y| ,\end{aligned} \tag{10}$$

with the substitutions

$$\begin{aligned}&\phi_{1Y}-\phi_{1X}\equiv\phi_1 ,\\ &\phi_{2Y}-\phi_{2X}\equiv\phi_2\end{aligned} \tag{11}$$

and with the assumption that there is no need to distinguish between $\mathcal{T}_x$ and $\mathcal{T}_y$ and between $\mathcal{R}_x$ and $\mathcal{R}_y$ at near normal incidence, this simplifies further and we have

$$\begin{aligned}\mathcal{P}_{12}(\theta_1,\theta_2)=\alpha_1\alpha_2\Big||\mathcal{T}|^2&[\cos(\theta_1-\chi_1)\cos(\theta_2-\chi_2)\cos\chi_1\sin\chi_2-\sin(\theta_1-\chi_1)\sin(\theta_2-\chi_2)\sin\chi_1\cos\chi_2e^{i(\phi_1+\phi_2)}\\
&+\cos(\theta_1-\chi_1)\sin(\theta_2-\chi_2)\cos\chi_1\cos\chi_2e^{i\phi_2}-\sin(\theta_1-\chi_1)\cos(\theta_2-\chi_2)\sin\chi_1\sin\chi_2e^{i\phi_1}]\\
-|\mathcal{R}|^2&[\cos(\theta_1-\chi_1)\cos(\theta_2-\chi_2)\sin\chi_1\cos\chi_2-\sin(\theta_1-\chi_1)\sin(\theta_2-\chi_2)\cos\chi_1\sin\chi_2e^{i(\phi_1+\phi_2)}\\
&-\cos(\theta_1-\chi_1)\sin(\theta_2-\chi_2)\sin\chi_1\sin\chi_2e^{i\phi_2}+\sin(\theta_1-\chi_1)\cos(\theta_2-\chi_2)\cos\chi_1\cos\chi_2e^{i\phi_1}]\Big|^2/(|\mathcal{T}|^4+|\mathcal{R}|^4) .\end{aligned} \tag{12}$$

Finally, we consider a 50%:50% beam splitter with $|\mathcal{T}|^2=\frac{1}{2}=|\mathcal{R}|^2$ and we let the phase plates be positioned symmetrically with $\chi_1=\chi=\chi_2$. Then Eq. (12) reduces to

$$\mathcal{P}_{12}(\theta_1,\theta_2)=\tfrac{1}{2}\alpha_1\alpha_2|\cos(\theta_1-\chi)\sin(\theta_2-\chi)e^{i\phi_2}-\sin(\theta_1-\chi)\cos(\theta_2-\chi)e^{i\phi_1}|^2 . \tag{13}$$

From Eq. (13) we immediately find ($\bar{\theta}\equiv\theta+\pi/2$)

$$\begin{aligned}\mathcal{P}_{12}(\theta_1,-)&=\mathcal{P}_{12}(\theta_1,\theta_2)+\mathcal{P}_{12}(\theta_1,\bar{\theta}_2)\\ &=\tfrac{1}{2}\alpha_1\alpha_2|\cos(\theta_1-\chi)\sin(\theta_2-\chi)e^{i\phi_2}-\sin(\theta_1-\chi)\cos(\theta_2-\chi)e^{i\phi_1}|^2\\ &\quad+\tfrac{1}{2}\alpha_1\alpha_2|\cos(\theta_1-\chi)\cos(\theta_2-\chi)e^{i\phi_2}+\sin(\theta_1-\chi)\sin(\theta_2-\chi)e^{i\phi_1}|^2=\tfrac{1}{2}\alpha_1\alpha_2 ,\end{aligned} \tag{14}$$

and similarly $\mathcal{P}_{12}(-,\theta_2)=\frac{1}{2}\alpha_1\alpha_2$, which implies that the photon in each arm is unpolarized. From this it follows that the probability for the detection of one photon in each arm is given by

$$\mathcal{P}_{12}(-,-)=\mathcal{P}_{12}(\theta_1,-)+\mathcal{P}_{12}(\bar{\theta}_1,-)=\alpha_1\alpha_2 \tag{15}$$

as expected. Hence

$$\begin{aligned}P_{12}(\theta_1,\theta_2)&\equiv\frac{\mathcal{P}_{12}(\theta_1,\theta_2)}{\mathcal{P}_{12}(-,-)}\\ &=\tfrac{1}{2}|\cos(\theta_1-\chi)\sin(\theta_2-\chi)e^{i\phi_2}\\ &\quad-\sin(\theta_1-\chi)\cos(\theta_2-\chi)e^{i\phi_1}|^2\end{aligned} \tag{16}$$

is the joint probability of detection normalized to the subspace of measured coincidences, given imperfect detectors. The probability is exactly the same for perfect detectors, if we are allowed to make the random (fair) sampling assumption, which has been shown to be consistent with numerous measurements.

It is worth noting that for two identical phase plates with $\phi_1=\phi_2=\phi$, Eq. (16) reduces to

$$P_{12}(\theta_1,\theta_2)=\tfrac{1}{2}\sin^2(\theta_2-\theta_1) , \tag{17}$$

which is of the usual form for a system exhibiting violations of locality. This result is independent of ϕ and χ and is exactly the same as if the two phase plates were not there at all. In general, however, $P_{12}(\theta_1,\theta_2)$ depends separately on θ_1 and θ_2.

III. TESTS OF THE BELL INEQUALITY

We shall make use of the Clauser-Horne type of Bell inequality for a system obeying local realism, which can be expressed in the form [13]

$$S\equiv P_{12}(\theta_1,\theta_2)-P_{12}(\theta_1,\theta_2')-P_{12}(\bar{\theta}_1',\theta_2)-P_{12}(\theta_1',\bar{\theta}_2')\le 0 , \tag{18}$$

whenever the single channel probabilities $P_1(\theta_1),P_2(\theta_2)$ are equivalent to the joint probabilities $P_{12}(\theta_1,-),P_{12}(-,\theta_2)$, respectively. This is certainly true for perfect detectors and it is true more generally if the fair sampling assumption is valid, so that the subensemble of detected photon pairs is representative of the whole ensemble of photon pairs. Here $\theta_1,\theta_1',\theta_2,\theta_2'$ are any four polarization angles, and $\bar{\theta}\equiv\theta+\pi/2$ is the polarization orthogonal to θ. With the particular choice

$$\begin{aligned}\theta_1&=\theta_0 ,\\ \theta_2&=3\pi/8+\theta_0 ,\\ \theta_1'&=3\pi/4+\theta_0 ,\\ \theta_2'&=\pi/8+\theta_0 ,\end{aligned} \tag{19}$$

and $P_{12}(\theta_1,\theta_2)$ given by Eq. (17), Eq. (18) yields the result

$$S=-\frac{1}{2}+\frac{1}{\sqrt{2}}\approx 0.207 \tag{20}$$

for all angles ϕ, χ, and θ_0, and this represents the greatest possible violation of the Bell inequality (18). It is not difficult to show from the more general relation (16) that

$$S=-\frac{1}{2}+\frac{1}{2\sqrt{2}}[1+\cos(\phi_2-\phi_1)] , \tag{21}$$

when $\phi_1\neq\phi_2$, which again violates the inequality so long as $|\phi_2-\phi_1|<66°$, and again does not depend on χ. On the other hand, if $\phi_1\neq\phi_2$ but $|\mathcal{T}|^2\neq|\mathcal{R}|^2$, it can be shown from Eq. (12) that S depends on χ also in general.

IV. EXPERIMENT

We have tested the validity of the inequality (18) by measuring the two-photon coincidence rate $R_{12}(\theta_1,\theta_2)$ with detectors D_1 and D_2 (see Fig. 1), which, after subtraction of accidental coincidences, is proportional to $P_{12}(\theta_1,\theta_2)$. We use the relations

$$P_{12}(\theta_1,\theta_2)=R_{12}(\theta_1,\theta_2)/R(-,-) , \tag{22}$$

where

$$\begin{aligned}R(-,-)&=R_{12}(\theta_1,\theta_2)+R_{12}(\theta_1,\bar{\theta}_2)\\ &\quad+R_{12}(\bar{\theta}_1,\theta_2)+R_{12}(\bar{\theta}_1,\bar{\theta}_2)\\ &=R_{12}(\theta_1,-)+R_{12}(\bar{\theta}_1,-)\\ &=R_{12}(-,\theta_2)+R_{12}(-,\bar{\theta}_2) .\end{aligned} \tag{23}$$

The polarization angles θ_1,θ_2 were adjusted, not by rotating the polarizer itself, but rather by rotating a halfwave plate in front of each polarizer by $\theta_1/2, \theta_2/2$. This ensured two things; (a) that the light falling on the photodetectors always had the same polarization as θ_1, θ_2 are varied and (b) that the beam was not deviated from measurement to measurement by the optically thick beam-splitting polarizers. Measurements of the parameter S were made for various values of χ from 0 to $\pi/4$ as shown in the plot of Fig. 2. The values of χ represent projections of the state onto polarization bases ranging from linear to circular.

The experimental results were obtained with a nominally 50%:50% BS, with $\chi_1 = \chi = \chi_2$ to an accuracy of 0.1° and with nominally identical quarter-wave plates (Q_1 and Q_2). The angles θ are those given by Eq. (19) for the special case $\theta_0 = 0$, and they were also set to an accuracy of 0.1°. It can be seen that S is positive and that the largest value $S = 0$ allowed by local realism is exceeded by each data point by about 40 standard deviations; it is exceeded by the average of all points by 100 standard deviations.

To a first approximation, S is independent of χ, as expected from Eq. (20). However, we also found that some dependence on χ may show up when the phase plates are translated at right angles to the light beams, so that the light passes through a different part of the plate. This is probably due to imperfections of the phase plates, and we suspect an effect of this sort may be at least partly responsible for some of the difficulties experienced in previous attempts to test local realism with this combination of polarizers and phase plates [7,8]. As our data points fall below the maximum value of 0.207 given by Eq. (20), it is possible that the phase plates Q_1, Q_2 may not have had the same phase delay. But in that case, $P_{12}(\theta_1, \theta_2)$ would depend upon χ rather than being constant as we found in the course of our measurements. It is also possible that an incomplete overlap of the two down-converted light beams due to dispersion or wave-front distortion within the nonlinear crystal may have played a role. Then the state of the photons would be a combination of the state in Eq. (1) and a diagonal density operator representing the nonoverlapping regions of the light beams, and this would modify the outcome.

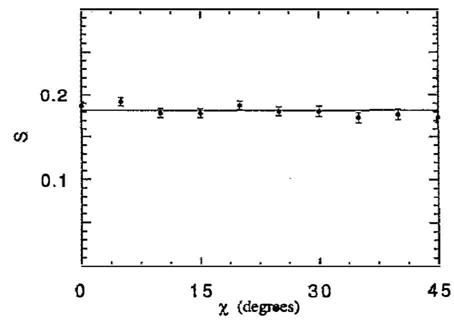

FIG. 2. Experimental results for the parameter S and its standard deviation in the Bell inequality for various settings χ of the quarter-wave plates.

ACKNOWLEDGMENTS

We are indebted to Dr. A. Garuccio for making us aware of the problems with previous experiments. This research was supported by the National Science Foundation and the U.S. Office of Naval Research. One of us (C.H.M.) acknowledges CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico, Brazil) for financial support.

[1] A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **47**, 460 (1981).
[2] A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **49**, 91 (1982).
[3] A. Aspect, P. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1992).
[4] Y. H. Shih and C. O. Alley, Phys. Rev. Lett. **61**, 2921 (1988).
[5] Z. Y. Ou and L. Mandel, Phys. Rev. Lett. **61**, 50 (1988).
[6] T. E. Kiess, Y. H. Shih, A. V. Sergienko, and C. O. Alley, Phys. Rev. Lett. **71**, 3893 (1993).
[7] J. F. Clauser, Nuovo Cimento **338**, 740 (1976).
[8] A. J. Duncan, H. Kleinpoppen, and Z. A. Sheikh, in *Bell's Theorem and the Foundations of Modern Physics*, edited by A. Van der Merwe, F. Selleri, and G. Tarozzi (World Scientific, Singapore, 1992), p. 161.
[9] J. Torgerson, D. Branning, and L. Mandel, Appl. Phys. B **60**, 267 (1995).
[10] J. R. Torgerson, D. Branning, C. H. Monken, and L. Mandel, Phys. Rev. Lett. (to be published).
[11] Z. Y. Ou, C. K. Hong, and L. Mandel, Opt. Commun. **63**, 118 (1987).
[12] See, for example, M. Born and E. Wolf, *Principles of Optics*, 6th ed. (Pergamon, Oxford, 1980), Sec 1.5.
[13] J. F. Clauser and M. A. Horne, Phys. Rev. D **10**, 526 (1974).